AP Worksheet #6 (Due: February 21st) – Reworked

End of Chapter 4

Note: This grade will go on the Second Semester.

Score: _________ Name: ______________________________

Section I – No calculators (Please show all work)

Grading Scale

(60 Points Possible)

A	= 52 correct	C+	= 44 correct
A-	= 50 correct	C	= 40 correct
B+	= 49 correct	C-	= 37 correct
B	= 47 correct	D+	= 36 correct
B-	= 45 correct	D	= 33 correct
		D-	= 30 correct
F	= 29 or less correct		

1. If $f(x)=2x^{1/4}$, then $f^{-1}(8)=$ ________
2. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{3x^3+2x+5}=$ __________
3. If $f(x)=\dfrac{2x+1}{x-1}$ then $f'(x)$ is ____________ (write as a single fraction)
4. If the function *f* is continuous for all real numbers and if $f(x)=\dfrac{x^2+x-12}{x+4}$ when $x\neq -4$, then $f(-4)=$ _________
5. If $x^3+4x^2y-3y^2=8$, then $\dfrac{dy}{dx}=$ ________________ (write as a single fraction)
6. If $f(x)=\tan x+\sec^2 x$, then $f'(x)=$ ____________________
7. An equation of a line normal to the graph of $y=3x^2+2x-1$ at (2,15) is _________________
8. $\int_{-1}^{1}\dfrac{4x}{(1+x^2)^2}\,dx=$ __________
9. If $f(x)=\tan^2 x$, then $f''(\pi)=$ _____________
10. If $f(x)=\dfrac{5}{x^2+1}$ and $g(x)=3x$, then $f(g(2))=$ ___________
11. $\int x\sqrt{5x^2-4}\,dx=$ ____________ (write using rational exponents)
12. The slope of the line tangent to the graph $3x^2+5y^2=17$ at (2,1) is ___________
13. The equation $y=1+5\sin\dfrac{\pi}{6}(x+5)$ has a fundamental period of _____________
14. For what value of *x* does the function $f(x)=2x^3-18x^2-240x$ have a local minimum? ___
15. If $f(x)=\begin{cases}x^2+5 \text{ if } x<2\\ 4x-5 \text{ if } x\geq 2\end{cases}$, for all real numbers *x*, which of the following must be true? Justify.

I. $f(x)$ is continuous everywhere.

II. $f(x)$ is differentiable everywhere.

III. $f(x)$ has a local minimum at $x=2$.

(A) I only (B) II only (C) II and III only

(D) III only (E) I, II, and III

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____.

16. The acceleration of a particle moving along the y-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at $t =$ ___________

17. The average value of a function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is ______

18. If $F(x) = \int \sqrt{(x^3 + 3x + 121)}(x^2 + 1)dx$ then $F(x) =$ ______________

19. $\lim\limits_{x \to 0} \dfrac{\sin 2x \cos x - \sin 2x}{x^2} =$ __________

20. If $f(x) = \tan^3(x + \pi)$, then $f'(\pi) =$ __________

21. $\int x\sqrt{x+3}\ dx =$ ___________

22. $\dfrac{d}{dx}\left[\int_2^{x^2} \ln(3t-5)dt\right] =$ _________

23. If a particle moves on a line according to the law $s = t^5 + 2t^3$, then how many times does it reverse directions? __________

24. A rectangular pigpen is to be built against a wall so that only three sides will require fencing. If p feet of fencing are to be used, the area of the largest possible pen is ___________.

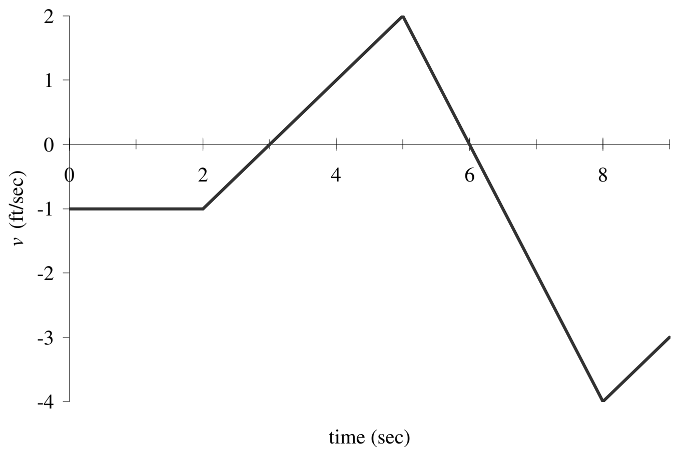
25. A smooth curve with equation $y = f(x)$ is such that its slope at each x equals x^2. If the curve goes through the point (-1,2), then its equation is ___________________.

26. If $G(2) = 5$ and $G'(x) = \dfrac{10x}{9 - x^2}$, then an estimate of G (2.2) using local linearization is approximately _____________.

27. The average value of $f(x) = 3 + |x|$ on the interval [-2,4] is _______________.

28. Suppose $f(x) = \dfrac{x^2 + x}{x}$, if $x \neq 0$ and $f(0) = 1$. Prove below that f is continuous at $x = 0$.

Section II (calculators may be used)



The graph shown is for questions 29 and 30. It shows the velocity of an object during the interval $0 \le t \le 9$.

29. The object obtains the greatest speed at $t =$ ______________.

30. The object was at the origin at $t = 3$. It returned to the origin at ________________________.

31. $\int_0^{\pi/4} \sin x \, dx + \int_{-\pi/4}^{0} \cos x \, dx =$ ___________

32. $\lim_{h \to 0} \dfrac{\sec\left(\dfrac{\pi}{6} + h\right) - \sec\left(\dfrac{\pi}{6}\right)}{h} =$ __________

33. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$ ______

34. If $f(x) = 3x^2 - x$, and $g(x) = x^2$, then $\int g(f(x))dx =$ __________

35. The graph of $y = 2x^3 - 5x^2 + x + 2$ has a local minimum at ___________

36. The average value of the function $f(x) = \dfrac{2x^2 - 3x + 1}{x - 1}$ on the interval [2,4] is __________

37. $\dfrac{d}{dx}\left(\int_0^{3x} \cos(t)dt\right) =$ ___________

38. If the definite integral $\int_1^3 (x^2 + 1)dx$ is approximated by using the Trapezoid Rule when $n = 4$, the error from the actual is ___________.

39. $\int(\cot^2 x)\,dx =$ __________

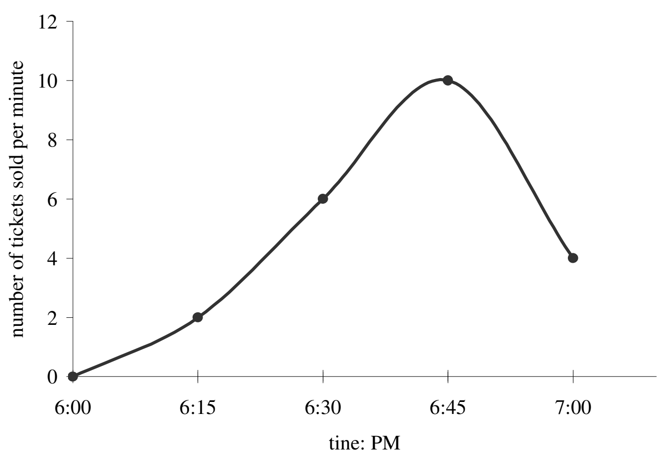
40. Find the distance traveled (to three decimal places) in the first 4 seconds, for a particle whose velocity is given by $v(t)=7\sin^2 t$; where t stands for time. __________

41. $\int \tan^6 x \sec^2 x\, dx =$ ___________

42. The intervals on which the function $f(x)=x^4-4x^3+4x^2+6$ increases are (is) __________.

43. If we replace $\sqrt{x-2}$ by u, then $\int_3^6 \frac{\sqrt{x-2}}{x}\,dx$ is equivalent to the integral _______________ (make sure the integral is in terms of u)

44. How many point of inflection does the function f have on the interval $0 \le x \le 6$ if $f''(x)=2-3\sqrt{x}\left(\cos^3 x\right)$? ________________



45. The graph shows the rate at which tickets were sold at a movie theater during the last hour before show time. Using right-rectangle method, an estimate of the size of the audience is ____________.

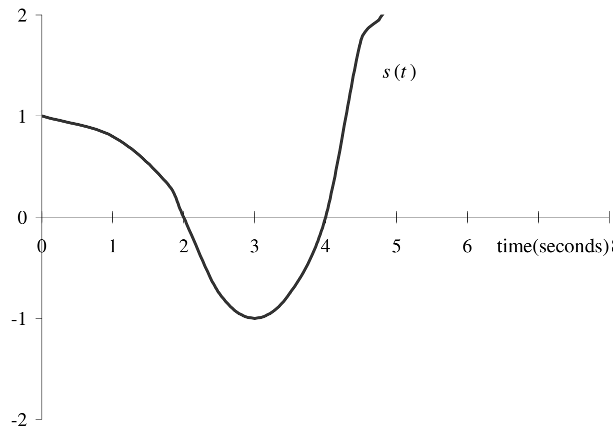
Section III Free Response Questions (No calculator) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

1) Let *f* be the function given by $f(x) = 1 + \frac{2}{x} + \frac{1}{x^2}$.

   a) Find the *x* and *y* intercepts. Justify.
   b) Write an equation for each vertical and horizontal asymptote for the graph of *f*. Justify.
   c) Find the intervals on which *f* is increasing and decreasing. Justify.
   d) Find the maximum and minimum values of *f*. Justify.

No Calculator – Work is to be shown on this page.

2) Let the graph of $s(t)$, the position function (in feet) of a moving particle, be given below. Let t be measured in seconds. The concavity changes at $t = 2$ and $t = 4$



a) Find the values of t for which the particle is moving to the right and when it is moving to left (i.e., when velocity is positive or negative, respectively). Justify.

b) Find the values of t for which the acceleration is positive and for which it is negative. Justify.

c) Find the values of t for which the particle is speeding up (i.e., when $|v|$ is increasing). Justify.

Section IV Free Response (calculator may be used) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

3) A particle moves along the *x*-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position $x(1) = 9$.

   a) Write an expression for the velocity of the particle $v(t)$.
   b) At what values of *t* does the particle change direction? Justify.
   c) Write an expression for the position function, $x(t)$, of the particle.
   d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 4$.

Calculator Allowed – Work is to be shown on this page.

4) A floodlight is on the ground 45 meters from a building. A thief 2 meters tall runs from the floodlight towards the building at 6 meters/second.
   a) Using a triangle(s) draw a picture of the situation.
   b) What is the relationship (equation) between the shadow on the building and the distance the thief is from the floodlight?
   c) How rapidly is the length of the shadow on the building changing when he is 15 meters from the building?